Constituent quark masses from modified perturbative QCD

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Abstract. A recently proposed modified perturbative expansion for QCD incorporating gluon condensation is employed to evaluate the quark and gluon self-energy corrections in first approximation. The results predict mass values of $1/3$ of the nucleon mass for the light quarks u, d, and s and a monotonously growing variation with the current mass. The only phenomenological input is that $\langle G^2 \rangle$ is evaluated up to order g^2 as a function of the unique parameter C defining the modified propagator, and then C is fixed to give a current estimate of $\langle g^2 G^2 \rangle$. The light quarks u and d as a result are found to be confined and the s, c, b and t ones show damped propagation modes, suggesting a model for the large differences in stability between the nucleons and the higher resonances. The above properties of quark modes diverge from the fully confinement result following from the similar gluon propagator previously considered by Munczek and Nemirovski. On the other hand, the condensate effects on the gluon self-energy furnish a tachyonic mass shell as predicted by the Fukuda analysis of gluon condensation in QCD.

1 Introduction

One of the great achievements of theoretical high energy physics in the last thirty years has been the discovery and development of quantum chromodynamics (QCD). The smallness of the coupling constant at high momenta (asymptotic freedom) has allowed one to develop a perturbative framework applicable for very high energy processes. That situation strongly simplifies the study of such phenomena and the calculated quantities are in good agreement with the experimental data. However, perturbative QCD (PQCD) is far from being able to furnish even a rough description for the relevant physics at low energies. The solution of this situation is currently one of the main challenges of particle physics.

A relevant phenomenon related with QCD is color confinement. Today, there exist strong reasons to believe that the relation between the basic quantities in QCD, like the gluon and quarks fields, and the real world characterized by a whole variety of interacting mesons and baryons, can be understood by solving the confinement problem. The basic picture which seems plausible, deriving from QCD, is that the fundamental fields cannot be associated to the asymptotic states of quark and gluons and the true physical states consist only of colorless composites of quarks and gluons (mesons and baryons).

The above quoted limitations of PQCD mean in particular that the usual Fock space vacuum of the non-interacting theory is unable to predict, even approxi-

mately, the real ground state properties of QCD [1–4]. This is in contrast with the case in QED, where the standard perturbation theory gives a better than good concordance with the experimental data. The nature of the structure of the vacuum is one of the main problems to be clarified and naturally its solution is closely linked with the color confinement effect. A short review of the various models which have been employed to investigate the confinement and the vacuum structure in QCD can be found in [5], and one may consult [6,7] for a more detailed analysis.

Another basic problem for QCD is the one related with the quark masses, that is, the so-called current quark masses which appear in the Lagrangian. As the quarks are not observed in free states, the meaning of the quark masses needs to be considered with care [8–10]. The nature of these parameters has been an important subject of research since a long time ago [11–13]. The relations among these masses and their magnitudes have been determined through the methods of current algebra in combination with measurements of the light mesons masses.

In former works [14,15,5] an attempt aimed at the construction of a modified perturbation expansion for QCD able to predict at least some low energy properties of this theory has been considered. First, in [14], the modified expansion conserving the color $SU(3)$ and Lorentz symmetries was proposed, aiming at the solution of the symmetry limitations of the earlier chromo-magnetic field models [1–4], inspired the search. A similar lack of manifest Lorentz

invariance had also the Feynman rules, intending to include gluon condensation, advanced later in [16–18], in which a delta function for $k < \Lambda_{\mathrm{QCD}}$ was summed over the perturbative piece. The expansion proposed in [14] considers a change of the gluon propagator in a term associated to condensation zero momentum gluons. This alteration of the usual rules had the interesting property of producing a non-vanishing value for the gluon condensation parameter $\langle g^2 G^2 \rangle$ already in the first approximation [19]. In addition, in [14] a non-vanishing value for the effective self-energy of the gluons was obtained at the loop level. Finally, a perturbative evaluation of the effective potential as a function of the condensate parameter indicated that the condensation is spontaneously generated from a zero condensate state.

The next work [15,5] had as a main objective the justification of the applicability of the Feynman expansion introduced in [14]. This search was in need because after modifying the propagator, it was unknown whether or not the initial wave function generating the expansion was a physical state of the free theory. In [15,5], by making use of the operational formulation of QCD developed by Kugo and Ojima [20], it was possible to find a physical state of the non-interacting theory, thus enabling one to generate through the Wick expansion the sort of propagators considered in [14]. The discussion allowed also a more precise characterization of the class of changes admitted in the diagrammatic expansion by the physical state condition on the initial state. Specifically, it was found that the C parameter, describing the gluon condensation, must be real and positive.

The present work is an expanded version of a letter sent for publication. Its main objective is to first evaluate $\langle G^2 \rangle$ in terms of the condensate parameter C up to order g^2, in order to further determine the quark masses after fixing the parameter $g^2 C$ to reproduce the accepted value for $\langle g^2 G^2 \rangle$. The essential result is the evaluation of the constituent masses for the light quarks. In order to perform this, we made use of the recent values reported for the current masses [21]. The idea inspiring this work was the possibility that the gluon condensation described by a modified expansion could predict the appearance of large masses for the light quarks (u, d and s) of the order of 1/3 of the nucleon mass (constituent quark mass), thus furnishing an explanation for the successful constituent quark models. Surprisingly, after fixing the condensate parameter $g^2 C$ to produce the most accepted value of the $\langle g^2 G^2 \rangle$ (through the evaluation here performed of this quantity in terms of $g^2 C$), it followed that the light quarks u, d and s got exactly the needed constituent-like values for the masses in one of the propagation modes of the modified mass shell. The free propagation modes, which are at discrepancy with the absence of free quarks, are here only arising within an approximation evaluating the main effects of the condensate. After the inclusion of all the terms completing the one-loop approximation in the gluon self-energy an interesting picture arises. It follows that the previously free propagation modes of the u and d quarks turn out to be confined. On the other hand, the s, c, b

and t quark waves become damped. That is, the squared rest masses get complex values. It should be stressed that the peaked structure of the propagator for the u and d at $(p^2)^{1/2} = m = 0.30915\,\mathrm{GeV}$ signals the value of m as the one expected to appear in the kinetic energy terms $\boldsymbol{p}^2/2m$ defining the free part of the bound state equations for the u and d constituted nucleons. Henceforth, a possible mechanism contributing to a justification of the drastic difference between the stability of nucleons and the higher resonances is suggested. The absence of propagating modes for the u and d might assure the absolute stability shown by the nucleons' bound states. Or otherwise, the presence of damped modes in the rest masses of the former quarks would contribute to the decaying channels observed for all the hadron resonances associated to these short lived states.

Next, after considering the ground state within each of the baryon resonance groups as classified in [21] and its set of three constituent quarks, and adding the calculated quark masses for each of them (which depends on the flavor through the assumed values of the current masses), a spectrum of baryon masses was estimated. The results again reasonably well reproduce the experimentally determined resonance energies [21]. This outcome suggests a less relevant role for the interaction energy among the quarks in determining the masses of these hadrons.

At this point it is important to make clear that earlier works [22,23] introduced a pure delta function at zero momentum in the gluon propagator, searching for a model of meson resonances. In these treatments the constants introduced, multiplying the delta function, were different from the one fixed in our previous work [15,5], leading also to very different results. In particular, the singularity structure of the quark propagator has no poles on the real p^2 axis [23] at variance with our results. This difference is directly connected with the definite sign for the coefficient of the δ-function which in our case was precisely determined in [15,5]. Since our analysis in that paper was based on the direct construction of the state incorporating the condensate, it follows that the difference with the results of [22,23] is rooted in a corresponding difference in the initial state determining the Wick expansion. It is also possible that both procedures lead to the same exact physical predictions, but the first approximations in each approach could turn out to be more appropriate for the discussion of specific physical issues. This question will be addressed elsewhere. It should further also be noticed that gluon condensate states reproducing the modified expansion of [22,23] have also been discussed in [24,25]. However, all these references are related with states showing confined quarks and massive gluons, at variance with our proposed state determining massive quarks (in the simplest approximation) and tachyonic gluons.

It also follows that another dispersion relation is obtained which predicts a vanishing value of the mass as $m_Q \to 0$. This mode could be connected with the family of low mass mesons (e.g. Π and K mesons). Specifically, the possibility is open that the bound states of quark excitations in these light mass states of the quarks could

describe such low lying mesons within the framework here presented. A circumstance pointing in this direction is that the spontaneous chirality breaking associated with this mass generation should be expected to be associated with some massless Goldstone modes. Therefore, it is possible that the bound states of such modes predicted by the Bethe–Salpeter equations turn out to describe the light mesons Π and K. This question however, will be deferred to a further study.

The calculations of $\langle G^2 \rangle$ and the quark and gluon masses will be performed up to order g^2. It seems worth anticipating that the approximation scheme will resemble a sort of quasi-classical limit in which the condensate seems to play the role of a macroscopic mean quantum field. Such an interpretation is in concordance with the initial motivation of our analysis as aimed to construct a covariant version of the chromo-magnetic models [14].

Another conclusion which should be noticed is that the evaluated gluon mass determined from the allowed values of the condensate parameter $g^2 C$ is tachyonic. The possibility for a tachyonic gluon mass has been recently discussed in the literature [26,27,18]. This result indicates that the free propagator of the expansion is not reproducing itself after the first approximation. This property, however, does not mean a limitation of the approach. This is because, at variance with [22,23], we are not assuming that the exact propagator has the same delta function structure. This form only arose for the free propagator as determined by the Wick expansion around the considered initial state.

This presentation is organized as follows: Sect. 2 is devoted to making precise the elements characterizing the modified perturbation expansion. In Sect. 3 the calculation of $\langle G^2 \rangle$ up to order g^2 for gluodynamics is presented, and the result is used to estimate the condensation parameter $g^2 C$. Further, in Sect. 4 the effects of the condensate on the quark masses are evaluated and the values obtained are used to estimate the spectrum of the ground states in each baryon group as classified in [21]. In Sect. 5 the calculation of the effect of the condensate on the one-loop self-energy for the gluons is evaluated. Finally, the results are summarized and some open questions and a conjecture are underlined in Sect. 6.

2 The modified Feynman rules

In this introductory section, the main elements and conventions of the modified expansion introduced in [14,15,5] will be reviewed. The generating functional of the Green functions in the modified theory has the form

$$Z\left[J, \bar{\eta}, \eta, \bar{\xi}, \xi\right] = \frac{1}{N} \int D\left(A^r, \bar{c}^r, c^r, \bar{\Psi}^r, \Psi^r\right)$$
$$\times \exp\left\{i \int d^4 x \mathcal{L}^{\text{sources}}\right\}, \quad (1)$$
$$\mathcal{L}^{\text{sources}} = \mathcal{L} + J^{\mu,a} A_\mu^{ra} + \bar{c}^{ra} \eta^a + \bar{\eta}^a c^{ra}$$
$$+ \bar{\Psi}^{ri} \xi_i + \bar{\xi}^i \Psi^{ri}, \quad (2)$$

$$N = \int D\left(A^r, \bar{c}^r, c^r, \bar{\Psi}^r, \Psi^r\right)$$
$$\times \exp\left\{i \int d^4 x \mathcal{L}\right\}, \quad (3)$$

where sources for all the renormalized fields $(A_\mu^{ra}, \bar{c}^{ra}, c^{ra}, \bar{\Psi}^{ri}, \Psi^{ri})$ have been introduced in the usual manner, and the effective Lagrangian for the bare fields was selected in the Kugo–Ojima quantization procedure for gauge theory:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{Gh} + \mathcal{L}_Q,$$
$$\mathcal{L}_G = -\frac{1}{4} F_{\mu\nu}^a(x) F^{\mu\nu,a}(x) - \frac{1}{2\alpha} \left(\partial^\mu A_\mu^a(x)\right)^2,$$
$$\mathcal{L}_{Gh} = -i\partial^\mu \bar{c}^a(x) D_\mu^{ab}(x) c^b(x),$$
$$\mathcal{L}_Q = \bar{\Psi}^i(x) \left(i\gamma^\mu D_\mu^{ij} - m_Q^* \delta^{ij}\right) \Psi^j(x).$$

The sum over the six quark flavors will be omitted everywhere in order to simplify the exposition. We think that no confusion should arise for this. The gluon field intensity has the usual form:

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g^* f^{abc} A_\mu^b(x) A_\nu^c(x),$$

where the D_μ^{ab}, D_μ^{ij} are the covariant derivatives in the adjoint and fundamental representations of the group $SU(3)$, respectively,

$$D_\mu^{ab}(x) = \partial_\mu \delta^{ab} - g^* f^{abc} A_\mu^c(x),$$
$$D_\mu^{ij}(x) = \partial_\mu \delta^{ij} - ig^* T^{ij,a} A_\mu^a.$$

Here, the bare fields and bare coupling constant and masses (g^*, m_Q^*), were expressed in terms of their renormalized counterparts following the same conventions as in [28]. Below, the renormalized masses and coupling will be denoted as m_Q and g respectively.

It should be recalled that as argued in [15,5] the physical predictions for the value $\alpha = 1$ of the gauge parameter should have physical meaning whenever the adiabatic connection of the interaction does not lead the evolving state out of the physical space. Therefore we will here fix this value of the parameter.

Then, after the standard procedure of extracting from the functional integral the exponential of the terms of higher than second order in the fields (vertices), the generating functional takes the form to be employed in the evaluations below [29]:

$$Z\left[J, \bar{\eta}, \eta, \bar{\xi}, \xi\right]$$
$$= N^{-1} \exp\left\{i\left[\frac{S_{abc}^G}{3! i^3} \frac{\delta^3}{\delta J_a \delta J_b \delta J_c} + \frac{S_{abcd}^G}{4! i^4} \frac{\delta^4}{\delta J_a \delta J_b \delta J_c \delta J_d}\right.\right.$$
$$\left.\left. + \frac{S_{ras}^{Gh}}{2! i^3} \frac{\delta^3}{\delta \bar{\eta}_r \delta J_a \delta(-\eta_s)} + \frac{S_{iaj}^Q}{i^3} \frac{\delta}{\delta \bar{\xi}_i \delta J_a \delta(-\xi_j)}\right]\right\}$$
$$\times Z_0\left[J, \bar{\eta}, \eta, \bar{\xi}, \xi\right], \quad (4)$$

with the free functional given by

$$Z_0\left[J, \bar{\eta}, \eta, \bar{\xi}, \xi\right] = \exp\left\{i\frac{J_a G_G^{ab} J_b}{2} + i\bar{\eta}_r G_{Gh}^{rs} \eta_s + i\bar{\xi}_i G_Q^{ij} \xi_j\right\},$$

and N normalizes Z to be one at vanishing sources. Use will be made of the compact DeWitt notation [30], in which a Latin letter, say $a, b, \cdots$, for a field symbolizes the space-time coordinates of it as well as all its internal quantum numbers. The same index in a source or a tensor symbolizes the same set of variables of the kind of fields associated with this specific index. For example $A_a = A^a_{\mu_a}(x_a)$, $c_a = c^a(x_a)$ and $\Psi^i = \Psi^i(x_i)$. Note that a particular convention has been also employed in which the same letter is employed for the global index (for all the coordinated and internal indices) and the one indicating the internal coordinates in the explicit form of the quantities. Such a procedure was useful for the calculations, and we also think this will not create confusion. As usual, repeated indices represent the corresponding space-time integrals and contracted, Lorentz, spinor or color components.

The following definitions also have been used in (4):

$$S^\alpha_{ijk\cdots} \equiv \left(\frac{\delta}{\delta\Phi_i} \frac{\delta}{\delta\Phi_j} \frac{\delta}{\delta\Phi_k} \cdots S^\alpha(\Phi) \right)_{\Phi=0},$$

for $\alpha = G, Gh, Q$ and $\Phi = A, \bar{c}, c, \bar{\Psi}, \Psi$,

$$G^{ij}_\alpha \equiv -S^{-1}_{\alpha,ij} \quad \text{for} \quad \alpha = G, Gh, Q,$$

where G, Gh and Q mean the gluon, ghost and quark parts of the action respectively.

There is only one main element determining the difference of the usual perturbative QCD expansion and the modified one considered in [14,15,5]. It is related with the form of the gluon free propagator. As proposed in [14] there is an additional term to the gluon free propagator which is absent in the standard expansion. In [5,15], such a term was shown to be a consequence of a Wick expansion based in a state constructed by acting on the usual vacuum with an exponential of pairs of zero momentum gluon and ghost creation operators. The ghost propagator [15,5] could remain unmodified if the parameters left after fixing the form of the wave function was taken real and positive [15,5]. Such a selection will be employed here. The quark propagator is not affected in any way by the gluon condensation as introduced in [14,15,5]. However, as will be commented on in the summary, the results of the present work led to the idea of a possible physical relevance of introducing quark condensates along similar lines as was done for the gluons. We consider the exploration of this possibility, which now is to be exposed, as one of the most interesting extensions of this work.

A technical point can be noticed for precision. The operator quantization employed in [15,5] takes the ghost fields to be satisfying the conjugation properties proper of the approach of Kugo and Ojima [20]. However, at the level of the Feynman diagram expansion the difference with the standard procedure is only a change of variables.

In accordance with the above remarks and the results of [14,15,5] the diagram technique defining the modified expansion has the following basic propagators for the renormalized fields:

$$G^{ab}_G = \delta^{ab} g^{\mu_a \mu_b} \left[\frac{1}{k^2 + i\varepsilon} - iC\delta(k) \right],$$

$$G^{rs}_{Gh} = \delta^{rs} \frac{(-i)}{k^2 + i\varepsilon},$$

$$G^{ij}_Q = \delta^{ij} \frac{m + p^\mu \gamma_\mu}{(m - p^2 - i\varepsilon)},$$

and the same normal and counterterm vertices as the usual one as in the conventions of [28]. The dimensional regularization and minimal subtraction (MS) is the renormalization procedure employed here.

Having defined the Feynman rules and the conventions to be used, below these will be applied to the calculation of various quantities of interest.

3 The mean value of G^2

The calculation of the mean value of the gluon field intensity squared is considered in this section. It extends the former evaluation done in [14] within a simpler approximation. The result will then be employed afterwards for fixing the value of the condensation parameter $g^2 C$ as the one necessary for furnishing the currently estimated value for $\langle g^2 G^2 \rangle$. The calculation will be performed up to order g^2 including the whole dependence on the condensate.

It is a helpful circumstance here that the one-loop renormalization procedure of the modified expansion can be performed identically as in the standard PQCD. This is a direct consequence of the fact that the δ-function kills the only existing integrals at one-loop level and therefore all the corresponding counterterms in the dimensional regularization are identical. This fact will be helpful in regularizing and renormalizing the composite operator mean value $\langle G^2 \rangle$.

For the evaluation of $\langle G^2 \rangle$ the following definition of this quantity will be employed:

$$\langle G^2 \rangle = \frac{\langle 0| S_g |0 \rangle}{\int dx^D} \tag{5}$$

$$= \frac{\int D\left(A^r, \bar{c}^r, c^r, \bar{\Psi}^r, \Psi^r\right) S_g[A^r] \exp\left\{i \int d^4 x \mathcal{L}^{\text{sources}}\right\}}{\int dx^D \int D\left(A^r, \bar{c}^r, c^r, \bar{\Psi}^r, \Psi^r\right) \exp\left\{i \int d^4 x \mathcal{L}\right\}},$$

where $S_g[A^r]$ represents the gluon part of the action in the absence of the gauge breaking term depending on the gauge parameter α, evaluated at the renormalized gluon fields A^r, that is

$$S_g[A^r] = -\frac{1}{4} \int d^4 x F^{ra}_{\mu\nu}(x) F^{r\mu\nu,a}(x).$$

Therefore, $\langle G^2 \rangle$ will be considered as defined by the mean value of the gluon action divided by the space-time volume in dimensional regularization.

After substituting in (5) the perturbative expression for the mean value in terms of the generating functional (4) it follows that

$$\langle 0| S_g |0 \rangle = \left\{ \left[\frac{S^g_{ab}}{2i^2} \frac{\delta^2}{\delta J_a \delta J_b} + \frac{S^g_{abc}}{3!i^3} \frac{\delta^3}{\delta J_a \delta J_b \delta J_c} + \frac{S^g_{abcd}}{4!i^4} \right. \right.$$

$$\left. \left. \times \frac{\delta^4}{\delta J_a \delta J_b \delta J_c \delta J_d} \right] Z\left[J, \bar{\eta}, \eta, \bar{\xi}, \xi\right] \right\}_{J, \bar{\eta}, \eta, \bar{\xi}, \xi = 0}. \tag{6}$$

Fig. 1. Diagram contributing to G_2 terms in the gluon condensate parameter

In the momentum representation the kernel S_{ab}^g has the form

$$S_{ab}^g = (2\pi)^4 \, \delta \left(k_a + k_b\right) \delta^{ab} \left[- \left(g_{\mu_a \mu_b} k_a^2 - k_{a,\mu_a} k_{a,\mu_a}\right)\right],$$

and the S_{abc}^g and S_{abcd}^g ones are the usual three and four leg gluon vertices [28].

Let us consider below the evaluation of G^2 up to order g^2. As any Feynman diagram can be expressed as a polynomial in the constant C, after associating a new diagrammatic line for this part of the free propagator, all the Feynman diagrams can be divided in two groups. The first, say G_1 including the diagrams not showing the new propagator line and another group, G_2, having one or more, we wil call "condensate" lines. Such lines have associated to them a multiplicative Dirac delta function at zero momentum.

After writing out all the expressions of type G_2 (up to second order in the renormalized coupling g^2) contributing to (6), it follows that the only non-vanishing diagram is the one appearing in Fig. 1 in which both gluon lines are substituted by "condensate" lines. The remaining terms vanish thanks to the zero momentum evaluation produced by the Dirac delta functions introduced by the "condensate" lines and also due to the vanishing of tadpoles in dimensional regularization. Also, it should be underlined that the retention of the iϵ factors defining the Feynman propagator, up to the evaluation of the integrals, was also important in receiving the zero result for some of the diagrams in G_2. The contribution related with Fig. 1 had been calculated before and the result is [14,15]

$$T_{G_2} = -\frac{1}{4} \frac{288 g^2 C^2}{(2\pi)^8}.$$

Next, the diagrams in G_1 are the ones contributing to G^2 up to order g^2 in standard perturbative QCD. They are illustrated in Fig. 2. The terms T_6 and T_7 represent the only non-vanishing and non-tadpole-like second order contributions. It can be noticed that the appearance of two tadpoles in the term T_7 leads to the exact vanishing of this contribution in dimensional regularization.

As for the diagram T_6: it corresponds to the expression

$$E_{T_6} = \frac{1}{3! \mathrm{i}^3} \int \frac{\mathrm{d}p^D}{(2\pi)^D \mathrm{i}} \frac{g^{\mu\nu} \delta^{ab}}{p^2 + \mathrm{i}\varepsilon} \Pi_{\mu\nu}^{(g)ab}(p),$$

where $\Pi^{(g)}$ is the gluonic contribution to the polarization tensor in second order, which has been explicitly calculated. This expression, after substituting for the known expression for Π and performing the Wick rotation, turns out to be of the form
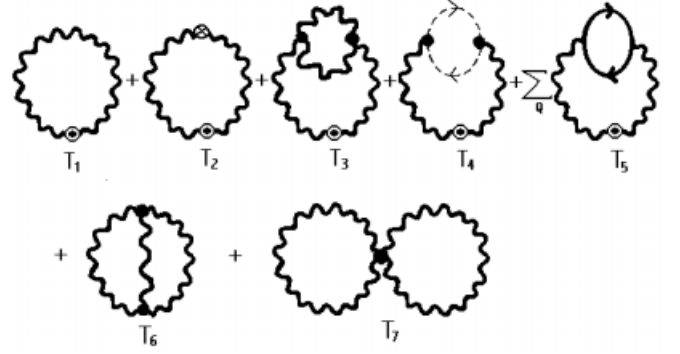

Fig. 2. Diagram contributing to G_1 terms in the gluon condensate parameter

$$E_{T_6} \backsim \int \frac{\mathrm{d}q^D}{(2\pi)^D} (q^2)^{(D/2)-2}. \tag{7}$$

After considering the formula [31]

$$\int \frac{\mathrm{d}q^D}{(2\pi)^D} \frac{1}{(q^2)^\alpha} \frac{1}{(q-k)^{2\beta}}$$
$$= \frac{\mathrm{i}(-1)^{-\alpha-\beta}(-k^2)^{-\alpha-\beta+n/2}}{(16\pi^2)^{n/4}} \frac{\Gamma(\alpha+\beta-n/2)}{\Gamma(\alpha)\Gamma(\beta)}$$
$$\times B\left(\frac{n}{2}-\beta, \frac{n}{2}-\alpha\right),$$
$$B(n,m) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)},$$

for the case $\alpha + \beta = 2 - (D/2)$ and fixing $k_\mu = 0$, follows the vanishing of diagram T_6 within the dimensional regularization for $D > 2$.

It remains now to discuss the contributions of the tadpole-like diagrams. It is worth noticing at this point that they are quite similar in form to the one corresponding to the square of the electromagnetic field strength in QED. In fact they differ from it only in some numerical color factors. An important quantitative difference between the expressions for both theories is related to the magnitude of the coupling constant which is very much higher in QCD. Let us consider the T_1–T_5 terms in Fig. 2 embodying the gluon, ghost and fermion contributions with the inclusion of the one-loop counterterms rendering finite the polarization operator at one loop [28]. This expression turns out to be of the form

$$E_{T_1-T_5} \backsim \int \frac{\mathrm{d}p^D}{(2\pi)^D} \frac{(p_\mu p_\nu - p^2 g_{\mu\nu}) \left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2 + \mathrm{i}\varepsilon}\right)}{p^2 + \mathrm{i}\varepsilon}$$
$$\times \delta^{aa}(1 - \Pi(p^2)), \tag{8}$$

where Π as usual is defined through the polarization tensor:

$$\Pi_{\mu\nu}^{ab}(p) = \delta^{ab}(p_\mu p_\nu - p^2 g_{\mu\nu}) \Pi(p^2),$$

and can be expressed as a superposition of the pure gluodynamical contribution and the quark one as [28]

$$\Pi(p^2) = \Pi_G(p^2) + \Pi_Q(p^2)$$

$$= \frac{g^2 \mu^{4-D}}{4\pi^{D/2}} (-p^2)^{2-D/2} \Gamma\left(2 - \frac{D}{2}\right) B\left(\frac{D}{2}, \frac{D}{2}\right)$$

$$\times \left(-\frac{C_G(3D-2)}{\frac{D}{2}-1} + \frac{8T_R}{B\left(\frac{D}{2}, \frac{D}{2}\right)} \sum_Q \int_0^1 dx \, x(1-x) \right.$$

$$\times \left. \left(x(1-x) - \frac{m_Q^2 - i\varepsilon}{p^2} \right)^{-(2-D/2)} \right) + (Z_3 - 1), \quad (9)$$

where $C_G = 3$ and $T_R = 1/2$ for $SU(3)$ and μ is the mass scale in the dimensional regularization.

Considering first the pure gluonic contributions associated with $(1 - \Pi_G(p^2))$ in (8), it follows that they reduce to a sum of dimensional regularized integrals of the same form as (7). Thus, the second order gluodynamical contribution of the tadpole-like contributions vanish. Let us analyze now the quark contributions associated to Π_Q. It can be noticed that for massless quarks, that is $m_Q = 0$, that the Π_Q has the same analytical dependence of p^2 as the gluonic part Π_G, and all the quark contributions vanish after the regularization is employed. However, for massive quarks the result for the second order dimensionally regularized contribution along the same lines employed for the previous terms gives rise to a non-vanishing contribution of the following form after the subtraction of the divergent pole terms at $D = 4$. The expression for the regularized term before subtraction is

$$g^2 \mu^{4-D} E_{T_6} = \left(\sum_Q (m_Q^2)^{D-2} \right)$$

$$\times \frac{64\sqrt{\pi} g^4 \mu^{8-2D} T_R(D-1)\Gamma(2-D)\Gamma(4-D)}{2^{7-2D}(4\pi)^D \Gamma\left(\frac{9}{2} - D\right)}.$$

This expression has simple as well as double poles which after being subtracted (in the $D = 4$ limit following the rules of the MS scheme for composite operators [32]) give a finite correction to $\langle g^2 G^2 \rangle$ for the gluon condensate parameter. It can be evaluated and the numerical expression becomes

$$\left(g^2 \mu^{4-D} E_{T_6}\right)^{\text{fin}}$$

$$= 0.00064 T_R g^4 \left(\sum_Q m_Q^4 \left(\left[55.32 - 37.4 \text{Log}\left(\frac{\mu^2}{m_Q^2}\right) \right. \right. \right.$$

$$+ \left. \left. \left. 6.\text{Log}^2\left(\frac{\mu^2}{m_Q^2}\right) \right] \right) \right).$$

However, it should be noticed that this finite expression has been obtained for the second order correction of the polarization operator. The summation of the infinite ladder of the self-energy insertion leading to the one-loop propagator can be suspected to produce a noticeable modification of this result. After also considering the high indefiniteness in the quark current masses, and

the technical difficulty we had in evaluating the dimensionally regularized expression for the above mentioned self-energy insertions, we decided to employ for the further consideration here the purely gluodynamical contribution. In support of this way of proceeding there is also another circumstance. One needs to notice that a refined version of the definition of the composite operator $G^2(x)$ could be suspected to exist in such a way that the terms associated to a tadpole diagram of the exact propagator vanish. This property would assure the exact vanishing of the $\langle F^2 \rangle$ in QED, a property that to our knowledge is expected. Because, at one loop, both QED and QCD tadpole expressions are basically the same, the above written quark contribution would not really appear. The existence of the mentioned definition, opens the possibility for the second order gluodynamical contribution to be exact for QCD in this approximation. However, we shift the discussion of this question to a further analysis, to be considered elsewhere, and simply adopt the gluodynamical result.

Therefore, the total second contribution to the mean value of G^2 in gluodynamics is defined by the only nonvanishing term

$$\langle G^2 \rangle = \frac{288 g^2 C^2}{(2\pi)^8},$$

coming from the condensate contributions arising from the diagram T_6 which is defined by the four gluon interactions.

Finally, let us determine in a phenomenological way a value for the constant $g^2 C$ by making use of a currently estimated value for $\langle g^2 G^2 \rangle$:

$$\langle g^2 G^2 \rangle \cong 0.5 \left(\text{GeV}/c^2 \right)^4.$$

Therefore, the parameter $g^2 C$ takes the value

$$g^2 C = 64.9394 \left(\text{GeV}/c^2 \right)^2. \quad (10)$$

This fixing of the condensation parameter allows one to investigate the physical predictions of the modified expansion in the next section.

4 Constituent quarks masses

Let us consider now the effect on the current quark masses produced by the condensate. For the full inverse propagator for the quarks in terms of the corresponding mass operator we have the general expression:

$$G_Q^{-1}(p) = (m_Q - p^\mu \gamma_\mu - \Sigma(p)), \quad (11)$$

where the color index identity matrix has been omitted and $\Sigma(p)$ is the mass operator. Its expression is related with the diagram in Fig. 3.

The form for $\Sigma(p)$ up to order g^2 within the modified Feynman rules can be written

$$\Sigma(p) = g^2 C_F \int \frac{d^4 k}{(2\pi)^4 i} \frac{\gamma_\mu (m_Q + (p-k)^\alpha \gamma_\alpha) \gamma_\nu G_0^{\mu\nu}(k)}{\left(m_Q^2 - (p-k)^2 \right)},$$

$$(12)$$

$$\sum{}^{ij} = \quad \text{(diagram)}$$

Fig. 3. Diagram for $\Sigma(p)$ in one loop

in which the color factor C_F for the $SU(N)$ group is $C_F = (N^2-1)/(2N)$ which for the case of interest here $SU(3)$, reduces to $C_F = 4/3$ [28].

Taking into account the modified gluon propagator and the standard relations for the γ matrices (which here are adopted in the same way as in [28]) the one-loop mass operator expression is simplified to

$$\Sigma(p) = g^2 C_F$$
$$\times \int \frac{d^4 k}{(2\pi)^4 \, i} \frac{2(2m_Q - (p-k)^\alpha \gamma_\alpha)}{\left(m_Q^2 - (p-k)^2\right)} \left(\frac{1}{k^2} - iC\delta(k)\right).$$

Let us disregard for the moment the term not involving the condensate parameter C in order to first study the sole influence on the quark masses of the condensation effect. Then the expression for Σ reduces to

$$\Sigma(p) = -\frac{g^2 C_F C}{(2\pi)^4} \frac{2(2m_Q - p^\alpha \gamma_\alpha)}{\left(m_Q^2 - p^2\right)}.$$

Therefore, the inverse propagator for the quark takes the simple form

$$G_Q^{-1}(p) = m_Q \left(1 + 2\frac{M^2}{\left(m_Q^2 - p^2\right)}\right)$$
$$- p^\mu \gamma_\mu \left(1 + \frac{M^2}{\left(m_Q^2 - p^2\right)}\right), \qquad (13)$$

where the constant M is defined by

$$M^2 = \frac{2g^2 C_F C}{(2\pi)^4} = 0.1111 \left(\text{GeV}/c^2\right)^2,$$

in which the numerical value has been obtained by using (10).

The zeros of the determinant associated to the inverse propagator (13) then allows us to determine the effects of the condensate (reflected by the parameter M) on the effective mass of the quarks. Let us below use s, where $s = 1, \cdots, 6$ is an index indicating each kind of the quark characterized by its particular current mass m_{Q_s}.

In order to find the dependence of the effective quark mass $m_{q_{s,i}}^2$ as a function of the current mass parameters m_{Q_s} an analysis was done of the zeros of the determinant of the inverse Green function (13). It follows that there is only one solution having a squared mass positive for arbitrary values of the current quark masses. The existence of this real and positive solution for the squared mass becomes possible thanks to the real and positive character of C, as was shown previously [15,5]. There is another interesting branch of the zeros of the determinant (dispersion
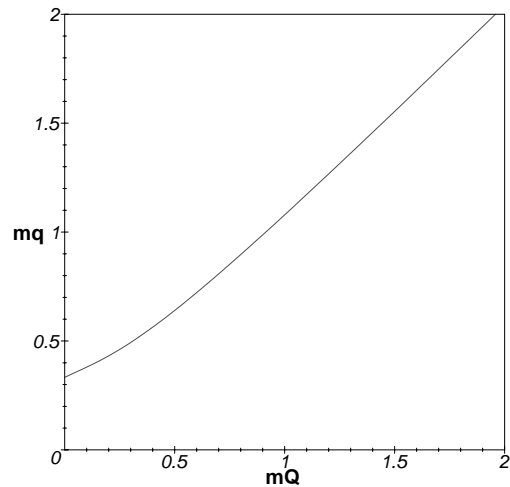

Fig. 4. Real solution for the quark mass as a function of the Lagrangian mass (masses in units of GeV/c^2)

Table 1. Quark mass values in presence of the condensate in units of MeV/c^2

Quarks	$m_{\text{Low}}^{\text{Exp}}$ [a]	$m_{\text{Up}}^{\text{Exp}}$ [b]	$m_{\text{Med}}^{\text{Theo}}$ [c]
(u)	1.5	5	334.944
(d)	3	9	336.287
(s)	60	170	388.191
(c)	1100	1400	1315.241
(b)	4100	4400	4269.572
(t)	168600	179000	173800.48

[a] Reported lower bound value for the Lagrangian mass
[b] Reported upper bound value for the Lagrangian mass
[c] Calculated mean value of the constituent mass, for the lower and upper bound of the Lagrangian masses as determined only by the condensate contribution to the self-energy in the one-loop approximation

relations) showing vanishing values for the effective mass in the light current mass limit. For the purely real solution the value of the effective quark mass $m_{q_s} = f(m_{Q_s})$ as a function of m_{Q_s} is shown in Fig. 4. The graph is plotted for the region $m_{Q_s} < 2 \, \text{GeV}/c^2$ which contains the current mass values of the u, d, s and c quarks.

As can be appreciated in the picture, the calculated effective quark masses for the light flavors (u, d and s) are clearly predicting the values of the quark masses in use in the constituent quark models of the baryons. That is, the light quarks get a weight of near one third of the nucleon mass. In Table 1 the mass values obtained for each of the quark flavors are shown [21]. From the global properties of Table 1 it can be observed that the main effect of the gluon condensate seems to be the dressing of the light quarks with a cloud of gluons having a total mass of one third of the nucleon mass. Then, the results point in the direction of the mainly glue nature of the constituent quark masses of the u, d and s quarks within many baryon resonances. These results support the idea that a modified perturbative expansion like the one considered, in which the effect

of the gluon condensate has been incorporated, would be able to predict with a reasonably good approximation the properties of the low energy strong interactions. Similar values for the constituent quarks masses have been obtained by different methods in [33,34].

We will now investigate the influence on the mass spectrum produced by including the standard one-loop contribution in the dispersion relations. After substituting the known [31] quark one-loop self-energy expression, the one-loop inverse quark propagator can be written

$$
\begin{aligned}
G_Q^{-1}(p) &= (m_Q - p^\mu\gamma_\mu - \Sigma(p)) \\
&= A(p^2)m_Q - B(p^2)p^\mu\gamma_\mu \\
&= m_Q\left[1 + \frac{4C_F g^2 C}{(2\pi)^4}\frac{1}{m_Q^2 - p^2} - \frac{C_F g(\mu)^2}{(4\pi)^2}\right. \\
&\quad\times\left(-6 + 4\gamma - \mathrm{Log}(4\pi) + 4\mathrm{Log}\left(\frac{m_Q^2 - p^2}{\mu^2}\right)\right. \\
&\quad\left.\left. - 4\frac{m_Q^2}{p^2}\mathrm{Log}\left(\frac{m_Q^2 - p^2}{m_Q^2}\right)\right)\right] \\
&\quad - p^\mu\gamma_\mu\left[1 + \frac{2C_F g^2 C}{(2\pi)^4}\frac{1}{m_Q^2 - p^2} + \frac{C_F g(\mu)^2}{(4\pi)^2}\right. \\
&\quad\times\left(1 - \gamma + \mathrm{Log}(4\pi) + \frac{m_Q^2}{p^2} - \mathrm{Log}\left(\frac{m_Q^2 - p^2}{\mu^2}\right)\right. \\
&\quad\left.\left. + \left(\frac{m_Q^2}{p^2}\right)^2\mathrm{Log}\left(\frac{m_Q^2 - p^2}{m_Q^2}\right)\right)\right],
\end{aligned}
\tag{14}
$$

where the dimensionless coupling constant g^2 has been substituted by its running expression as a function of the scale parameter μ so that

$$
g(\mu)^2 = \frac{4\pi 0.12}{1 + \dfrac{7\times 0.24}{4\pi}\mathrm{Log}\left(\dfrac{\mu}{91.18}\right)}.
$$

That is, the value of $\alpha(\mu)^2 = g(\mu)^2/(4\pi)$ is fixed at 0.12 at the scale of the Z mass $\mu = m_Z = 91.18\,\mathrm{GeV}$. In order to automatically fix the scale μ to the one associated to the magnitude of the obtained solution for the mass, the value of μ was taken to satisfy $\mu^2 = p^2$ to reproduce the same value of the solution for the squared mass.

Before proceeding, it should be noticed that the inverse propagator, after the introduction of the one-loop terms, gets a branch cut at the real axes for the considered complex variable p^2 for $\mathrm{Re}(p^2) > m_Q^2$. However, the $i\epsilon$ prescription selects one of the sides of the cut as giving a well-defined value of the inverse propagator at real p^2 values.

After multiplying (14) by $A(p^2)m_Q - B(p^2)p^\mu\gamma_\mu$ the modified mass shells can be most easily found by solving

$$
D(q^2) = A(p^2)^2 m_Q^2 - B(p^2)^2 p^2 = 0.
$$

Concretely, we estimated the corrected values of the masses in Table 1 by searching for the peaks of the inverse

Table 2. Quark mass values in presence of the condensate in units of MeV/c^2

Quarks	$m_{\mathrm{Low}}^{\mathrm{Exp}\,a}$	$m_{\mathrm{Up}}^{\mathrm{Exp}\,b}$	$m_{\mathrm{Med}}^{\mathrm{Theo}\,c}$
(u)	1.5	5	309.15
(d)	3	9	309.15
(s)	60	170	354.00
(c)	1100	1400	1343-200i
(b)	4100	4400	4515-65i
(t)	168600	179000	190000-1450i

[a] Reported lower bound value for the Lagrangian mass
[b] Reported upper bound value for the Lagrangian mass
[c] Calculated mean value of the constituent mass, for the lower and upper bound of the Lagrangian masses after the inclusion of the standard one-loop correction for the self-energy

for the absolute value of $D(q^2)$ as a 2D-function of the real and imaginary parts of p^2. The new values are reported in Table 2. As can be observed the masses were not drastically changed. They all show a relatively small reduction in their magnitude, and indicate the consistency of the predicted constituent masses for the light quarks at the one-loop level.

An interesting outcome following from the inclusion of the standard one-loop terms should be stressed. It consists in the smallest m_Q masses being associated to the lower and higher bounds for the u and d quarks, $1/\mid D(q^2)\mid$, although showing a definite peak at $0.30915\,\mathrm{GeV}$ does not really diverge near this value. Therefore, the propagator is not showing a definite pole associated to a propagating quark mode at this p^2 value. However, at the level of the Bethe–Salpeter equation for bound states of quarks, it should be expected that p^2 at this peak should behave as the non-relativistic value for the square of the mass entering the kinetic energy expression $p^2/2m_Q$. Henceforth, in spite of the presence of free propagating modes in the simplest approximation, the inclusion of the quantum one-loop correction makes the u and d modes non-propagating (confined) in the same way as occurs in the approach of Munczek and Nemirovsky [22,23].

For greater values of m_Q in the range presently estimated for the s quark current mass $(0.60\text{–}0.170\,\mathrm{GeV})$ in the neighborhood of the non-diverging peak of $1/\mid D(q^2)\mid$, this quantity starts showing divergence points at complex values of p^2 which represent damped but propagating quark modes. The structure of the spectrum just described suggests a possible explanation for the drastic difference of stability between the u and d composed hadrons and the resonances including s, c, b and t quarks. The point is that the presence of poles at complex p^2 for the heavier quarks could lead to the existence of disintegration channels for the higher mass resonances in which the s, c, b and t quarks can escape in a damped propagating mode to be hadronized by some separate mechanism. On the other hand, as the u and d quarks do not show propagating states, those could remain in perennial confinement within an undisturbed nucleon. A further study

Table 3. Experimental and theoretical baryonic resonance masses in units of MeV/c^2

Baryon	Exp.Val.	Th.Mean.Val.	Rel.Err.
$p(uud)$	938.27231	1006.175	7.24
$n(udd)$	939.56563	1007.519	7.23
$\Lambda(uds)$	1115.683	1059.422	5.04
$\Sigma^+(uus)$	1189.37	1058.078	11.04
$\Sigma^0(uds)$	1192.642	1059.422	11.17
$\Sigma^-(dds)$	1197.449	1060.766	11.41
$\Xi^0(uss)$	1314.9	1111.325	15.48
$\Xi^-(dss)$	1321.32	1112.669	15.79
$\Omega^-(sss)$	1642.45	1164.572	29.10
$\Lambda_c^+(udc)$	2284.9	1986.472	13.07
$\Xi_c^+(usc)$	2465.6	2038.375	17.33
$\Xi_c^0(dsc)$	2470.3	2039.719	17.43
$\Omega_c^0(ssc)$	2704	2091.622	22.65
$\Lambda_b^0(udb)$	5624	4940.803	12.15

Table 4. Experimental and theoretical masses for a group of vector mesons in units of MeV/c^2

Meson	Exp.Val.	Th.Mean.Val.	Rel.Err.
$\rho\left(\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}\right)$	770.0	671.231	12.83
$\varpi\left(\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}\right)$	781.94	671.231	14.16
$\phi(s\bar{s})$	1019.413	776.381	23.84
$J/\psi(1S)(c\bar{c})$	3096.88	2630.482	15.06
$Y(1S)(b\bar{b})$	9460.37	8539.144	9.74

of the consistency of this explanation is expected to done be within the planned extension of this work to investigate bound state equations for hadrons.

It can be concluded that the constituent quark masses evaluated from the simplest approximation for the dispersion relation considered above are stable under the inclusion of the standard one-loop corrections to the self-energy.

Next, in Table 3, the masses of the ground state resonances, each of them selected from one of the groups of baryons as classified in [21], are shown in comparison with the values obtained for them by a theoretical estimate consisting in adding the masses calculated here for each of their known quark constituents. The quark masses of Table 1 were employed for this evaluation. Also, in Table 4, the masses for some vector mesons reported also in [21] are shown in conjunction with the result of the sum of the calculated masses for their constituent quarks (Table 1).

As can be appreciated, the results obtained for the vector mesons and baryon resonances are only estimates. However, the values obtained are in reasonable correspondence with the reported experimental values. It can also be noticed that the validity of the present calculations implies that the binding energy contribution to the baryon
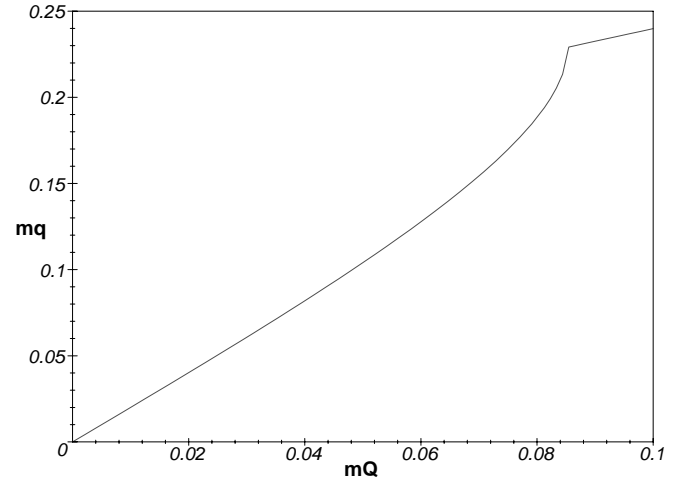
Fig. 5. A solution for the quark mass as a function of the Lagrangian mass, that satisfies $m_{q_s} \to 0$ when $m_{Q_s} \to 0$ (masses in units of GeV/c^2)

rest masses should be small. This property then could justify a non-relativistic description of such resonances.

As was remarked before, another form of the dependence of the effective quark mass with m_{Q_s} also follows from the dispersion relations. It by contrast shows a vanishing effective mass when $m_{Q_s} \to 0$. The existence of this branch opens the possibility of an appropriate description of the light mesons (Π and K) within the proposed scheme. They could be associated to bound states of light quarks propagating in such modes. The analysis of this question deserves a separate treatment and will be examined elsewhere. The graphic for the mentioned branch is represented in Fig. 5, for the region of $m_{Q_s} < 0.1\,\text{GeV}/c^2$, by a point at which the change in the analytic dependence signals where the solution gets an imaginary part when the current mass value m_Q grows.

5 Tachyonic gluon self-energy

Let us consider now the evaluation of the gluon self-energy. As is known the exact gluon propagator can be written in the form

$$G_G^{ab}(p) = \frac{\delta^{ab}}{p^2}\left(\frac{g_{\mu_a\mu_b} - \frac{p_{\mu_a}p_{\mu_b}}{p^2}}{1 + \Pi(p^2)} + \alpha\frac{p_{\mu_a}p_{\mu_b}}{p^2}\right), \quad (15)$$

where the function Π is defined at the one-loop level, by the expression

$$\Pi^{ab}(p^2) = \Pi_G^{ab} + \Pi_T^{ab} + \Pi_{Gh}^{ab} + \Pi_Q^{ab}$$
$$= \delta^{ab}\left(p_{\mu_a}p_{\mu_b} - g_{\mu_a\mu_b}p^2\right)\Pi(p^2),$$

which is diagrammatically represented in Fig. 6. Again, expanding in powers of the condensate parameter C, the total contribution can be decomposed in the standard one (associated to the ghost and the quark loop integrals) and

Fig. 6. Diagrams for $\Pi^{ab}\left(p^2\right)$

the C dependent ones Π_G^{ab} and Π_T^{ab} which can be readily calculated to be

$$\Pi_G^{ab} = \frac{g^2 C N}{(2\pi)^4}\delta^{ab}\left(-5g_{\mu_a\mu_b} + 2\frac{p_{\mu_a}p_{\mu_b}}{p^2}\right),$$

$$\Pi_T^{ab} = \frac{3g^2 C N}{(2\pi)^4}\delta^{ab}g_{\mu_a\mu_b}.$$

Summing these particular two terms leads to

$$\Pi_C^{ab}\left(p^2\right) = \delta^{ab}\left(p_{\mu_a}p_{\mu_b} - g_{\mu_a\mu_b}p^2\right)\frac{2g^2 C N}{(2\pi)^4 p^2}.$$

Considering again our case, the $SU(3)$ group, that is $N = 3$, and disregarding in a first consideration the standard one-loop contribution, the function Π reduces to

$$\Pi\left(p^2\right) = \frac{6g^2 C}{(2\pi)^4}\frac{1}{p^2}, \qquad (16)$$

which in turns leads to the following condition for the poles of (15):

$$p^2 - m_G^2 = 0,$$

where

$$m_G^2 = -\frac{6g^2 C}{(2\pi)^4} = -0.25\left(\text{GeV}/c^2\right)^2.$$

Therefore, as the parameter C has been defined before as real and positive [15,5], it follows that the transverse gluon mass correction becomes tachyonic. The ability of a tachyonic mass in producing improvements in models for the inter-quark potential has recently been argued in the literature [26,27]. The effect is linked with the introduction of a linearly increasing term as a first correction to the Coulomb potential in the massless case [27]. Therefore, the tachyonic result arising here for the gluon mass appears to be of interest for a derivation, within the present framework, of former successful phenomenological bound state models for mesons [35–37].

We are grateful to J.P. Vary and J. Qiu for the suggestion of this possibility.

It should also be underlined that the tachyonic character of the gluon self-energy arising here is fully compatible with the more general result of Fukuda for the normal Green functions in QCD, when gluon condensation is present [38]. This circumstance then suggests that the modified expansion under discussion could be a sort of perturbative representation of the Fukuda non-perturbative treatment of the gluon condensation in QCD. In a different manner, the approach of Celenza and Shakin and other

authors can be expected to be related with a different initial state also incorporating gluon condensation. It could be the point that, in the end, both pictures can appear to be equivalent at the level of exact results. This could be the case because the physical degrees of freedom are not the gluons and quarks ones, but their colorless composite combinations the physics of which is determined by the bound state dispersion relations.

Finally in this section it is to be stressed that if a modification of the ghost propagator is present, then a mass terms for the longitudinal propagator could also appear and the polarization operator (gluon self-energy) becomes non-transverse. This result means a violation of a Ward identity representing the gauge invariance by the presence of ghost (and fermion) condensates. Thus, it follows that the selection of the parameter of the initial state in [15,5] was the appropriate one for the satisfaction of a gauge invariance condition at this stage. The physical meaning of the alternative procedures of constructing the initial state needs however clarification.

6 Summary

The following results can be enumerated.

(1) A calculation of the mean value of G^2 up to order g^2 in gluodynamics was performed. The result coincided with a previous one obtained in a simpler approximation [14,15], which signals that the quartic gluon self-interaction is relevant in fixing a non-vanishing gluon condensation parameter already in the first order of the perturbation theory [19].

(2) The one-loop corrections for the quark masses under the influence of the condensate were evaluated. When only the terms reflecting the effect of the condensate are considered, the results for the constituent masses showed a branch which is real as a function of all the values of the quark current mass. Another solution which is real up to a critical value of the Lagrangian masses, and having a vanishing proper mass in the limit of small current masses is also obtained. The former branch predicts values of one third of the nucleon mass for the masses of the light quarks. The effective quark masses for all the quark flavors were evaluated as determined by their known current mass in the mentioned approximation.

(3) After the inclusion of the rest of the terms completing the full one-loop contribution to the quark self-energy, an interesting outcome arises. The u and d quarks, although showing high peak values of the propagator components near the former values of the squared proper masses, become confined. That is, their propagators do not show pole structures. On the other hand the s, c, b and t quarks acquire poles but now not at real values of p^2. Therefore, these free modes are damped. These properties suggest the possibility of the explanation for the disparity of the stability properties of the nucleons and higher resonances. It would work as follows: the absence of propagation solutions

for the u and d quarks could assure the unlimited life time of nucleons. Moreover, the damped oscillations present for the rest of the quarks can create decay modes for the bound states composed of them.

(4) By means of the obtained mass values, the ground state energies selected within each of the groups of hadron resonances reported in [21] were estimated through the simple addition of the masses of their known quark constituents. It could be remarked that the obtained values are reasonable in comparison with the data, after taking into account the high errors in the experimentally determined Lagrangian masses. This fact supports the hypothesis about the main glue nature of the constituent mass values of the light quarks. The small contribution of the binding potential energy of such resonances to their total mass is also indicated and thus suggests the applicability of non-relativistic approximations in studying them.

(5) The contribution of the condensate to the gluon self-energy of order g^2 was calculated. A tachyonic result for the gluon mass arises for the parameter values employed in the construction of the initial state defined in [15,5]. This conclusion is in concordance with the analysis of Fukuda for gluon condensation [38]. As was remarked before, there are recent arguments claiming the possibility of a tachyonic gluon mass [26,27,18] and its role in improving the quark–antiquark potential in bound state models of mesons [27].

It seems useful to comment here on some basic issues related with the modified expansion being considered.

(a) The Feynman expansion depending on the condensate parameter and taken for the gauge parameter $\alpha = 1$ corresponds with the Wick expansion around a physical state of the interaction free QCD. In this somewhat limited way the arbitrariness of the gauge parameter in the result for the gluon mass in [14] was solved in [15,5]. However, in order to furnish a fully consistent picture including renormalization, a physically equivalent version of the expansion for arbitrary values of α should be developed. This question will be addressed in future works.

(b) In [15,5] the condensation parameter C was defined as a real and positive one. This result determined the tachyonic character for the gluon mass found here. Further, the sign of C also fixes the real values of the order of the one third of the nucleon mass for the constituent quark masses also calculated in the present work. The opposite sign produces confined quark modes.

(c) An aspect which is important to underline is that the selection of the parameters defining the initial state in [15,5] was designed also in order to impose the absence of a modification for the free ghost propagator. However, as followed from the present analysis, this property in turn is related with the fact that a condensate modification of the ghost propagator can produce a longitudinal contribution to the self-energy. But such a term should not exist for the transversality condition of the polarization operator (a Ward

identity) to be obeyed. Thus, its appearance could manifestly break the gauge invariance. Therefore, the present work has also given a foundation to the non-modified ghost propagator choice considered in [14, 15,5].

At the present stage we identify some important questions to be addressed in a further extension of this work.

(1) To explore the possibility of showing the exact gauge parameter independence of the physical results within the modified expansion.

(2) To address the general proof of the renormalizability in the expansion.

(3) To search for a derivation of existing successful bound state models for the heavy quark mesons [35–37], as realized by the ladder approximation for the Bethe–Salpeter equation within the proposed modified expansion. The presence of a tachyonic gluon propagator in the approach (which is argued to have the effect of introducing a linearly increasing component in the inter-quark potential [27]) and the obtained constituent values for the light quark masses already support such a possibility.

(4) To improve the study of the effective action as a functional of the condensate parameter as done in [14] by also including the gauge field in order to search for a variant of the leading logarithm model useful for the investigation of field configurations associated to inter-quark strings, nucleon–nucleon potentials, etc.

(5) It can be underlined that the considered expansion seems to be able to furnish a general way for the perturbative description of pair condensates in many body theory. An important example coming to mind is the BCS state of the usual superconductivity. Pair states being more closely connected with QCD were discussed in [39,40]. In addition, in [40] it was advanced before the idea of using diquark condensates as generating an alternative for the Higgs mechanism.

Finally, we would like to make a conjecture about a possibility suggested by the results of this work. It is related with the question of whether the mass spectrum of the whole three generations of fundamental fermions could be predicted by a slight generalization of the modified perturbation expansion under consideration. In this sense, the given arguments led us to the idea that (after the introduction of quark condensates along the same lines as was done for gluon ones in [14,15,5]) the perturbation expansion obtained can have a chance in predicting both the Lagrangian mass and the constituent quark mass spectra of the three families of fundamental fermions. The fermion condensates as described in the proposed perturbative way would have the role of producing the Lagrangian quark masses, through the chiral symmetry breaking. The gluonic condensates, on the one hand, and as illustrated here, could be responsible of generating states of large constituent mass for the low mass quarks states. On the other hand, it seems feasible that the higher order radiative corrections (including color interactions with the condensate as well as chiral symmetry corrections) could also determine the mass spectra for

leptons and neutrinos. The smaller scale for the masses of these particles could be produced by the lack of either color or electric-weak lower order interaction terms in their self-energy. Therefore, the possibility that the Lagrangian mass spectrum of the three generations of the fundamental fermions could be predicted by a modified perturbation expansion of the sort being proposed is suggested. Work directed to investigate this possibility will be considered elsewhere. The modified expansion under study could be a useful technique for implementing this idea.

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